

What is claimed is:

- 1 1. A method for reconstructing an image of a scattering medium, comprising:
2 directing energy into the scattering medium at a source location on the
3 scattering medium;
4 measuring the energy emerging from the scattering medium at a detector
5 location on the scattering medium;
6 selecting an initial guess of internal properties of the scattering medium;
7 predicting the energy emerging from the scattering medium using an
8 equation of radiative transfer, wherein the prediction is a function of the initial guess;
9 generating an objective function based on a comparison of the prediction
10 with the measurement;
11 generating a gradient of the objective function by a method of adjoint
12 differentiation;
13 modifying the initial guess of the properties of the scattering medium
14 based on the gradient of the objective function; and
15 generating an image representation of the internal properties of the
16 scattering medium.

- 1 2. The method according to claim 1, further comprising repeating the
2 predicting of the energy emerging from the scattering medium based on the modified
3 initial guess, generating the objective function and modifying the initial guess, until at
4 least one of a predetermined number of repetitions has occurred and the objective
5 function reaches a predetermined threshold.

1 3. The method according to claim 1, wherein the prediction depends on the
2 boundary conditions.

1 4. The method according to claim 3, wherein the boundary conditions
2 account for a refractive mismatch at an interface between the medium and at least one of
3 the detectors and source.

1 5. The method according to claim 1, wherein the prediction comprises an
2 iterative process producing intermediate results.

1 6. The method according to claim 5, wherein the intermediate results of the
2 prediction are stored.

1 7. The method according to claim 6, wherein generating the gradient of the
2 objective function by adjoint differences uses the intermediate results of the prediction.

1 8. The method according to claim 7, wherein generating the gradient
2 comprises stepping backward through the intermediate results of the prediction.

1 9. The method according to claim 1, wherein the equation of radiative
2 transfer is time independent.

1 10. The method according to claim 9, wherein the time independent equation
2 of radiative transfer is:

3
$$\omega \nabla \Psi(\mathbf{r}, \omega) + (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega) = S(\mathbf{r}, \omega) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega') d\omega'$$

4 where $\Psi(\mathbf{r}, \omega)$ is the radiance at the spatial position \mathbf{r} directed into a unit
5 solid angle ω , $S(\mathbf{r}, \omega)$ is the energy directed into the medium at spatial position \mathbf{r} into a
6 unit solid angle ω , μ_s is the scattering coefficient, μ_a is the absorption coefficient and
7 $p(\omega, \omega')$ is the scattering phase function.

1 11. The method according to claim 10, wherein the scattering phase function
2 is:

3
$$p(\cos \theta) = \frac{1 - g^2}{2(1 + g^2 - 2g \cos \theta)^{3/2}}$$

4 where θ is the angle between the two unit solid angles ω and ω' , and g is
5 the anisotropy factor.

1 12. The method according to claim 1, wherein the equation of radiative
2 transfer is time dependent.

1 13. The method according to claim 12, wherein the time dependent equation
2 of radiative transfer is:

3
$$\frac{1}{c} \frac{\partial \Psi(\mathbf{r}, \omega, t)}{\partial t} = S(\mathbf{r}, \omega, t) - \omega \cdot \nabla \Psi(\mathbf{r}, \omega, t) - (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega, t) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega', t) d\omega'$$

4 where $\Psi(\mathbf{r}, \omega, t)$ is the radiance at the spatial position \mathbf{r} directed into a unit
5 solid angle ω , $S(\mathbf{r}, \omega, t)$ is the energy directed into the medium at spatial position \mathbf{r} into a

6 unit solid angle ω , μ_s is the scattering coefficient, μ_a is the absorption coefficient and
7 $p(\omega, \omega')$ is the scattering phase function.

1 14. The method according to claim 13, wherein the scattering phase function
2 is:

3
$$p(\cos\theta) = \frac{1 - g^2}{2(1 + g^2 - 2g\cos\theta)^{3/2}}$$

4 where θ is the angle between the two unit solid angles ω and ω' , and g is
5 the anisotropy factor.

1 15. The method according to claim 1, wherein the properties include at least
2 one of a scattering coefficient, an absorption coefficient, an anisotropy factor, and a
3 scattering phase function.

1 16. The method according to claim 1, wherein the objective function is a
2 normalized comparison of the predicted energy and the measured energy

1 17. The method according to claim 1, wherein the objective function is based
2 on the normalized sum of the differences between the predicted energy and the measured
3 energy for each source detector pair, wherein a source detector pair is formed between
4 each source location and each detector location.

1 18. The method according to claim 1, wherein the objective function is:

2
$$\phi = \frac{1}{2} \sum_i^m (P_i - M_i)^2$$

3 where \mathbf{M}_i represents the actual measurements and the \mathbf{P}_i represents the
4 predicted measurements for each source defector pair i , m is the number of source
5 detector pairs, where a source detector pairs is formed between each source location and
6 each detector location.

1 19. The method according to claim 1, further comprising minimizing the
2 objective function.

1 20. The method according to claim 19, wherein minimizing the objective
2 function includes a one dimensional line search.

1 21. The method according to claim 20, wherein the one dimensional line
2 search is performed along a direction of the gradient of the objective function.

1 22. The method according to claim 20, wherein the one dimensional line
2 search is performed along a gradient-dependent direction.

1 23. The method according to claim 1, wherein the energy comprises near
2 infra-red energy.

1 24. The method according to claim 1, wherein the scattering medium contains
2 regions wherein the scattering coefficients are not substantially greater than the
3 absorption coefficients.

1 25. The method according to claim 1, wherein the scattering medium contains
2 a low scattering region embedded in a high scattering region.

1 26. The method according to claim 1, wherein the predicted energy is
2 determined using finite element methods.

1 27. The method according to claim 1, wherein the predicted energy is
2 determined using finite difference methods.

1 28. A method for imaging the spatial optical properties of tissue, comprising:
2 (a) directing energy into the scattering medium at a source location on
3 the tissue;
4 (b) measuring the energy emerging from the scattering medium at a
5 detector location on the tissue;
6 (c) selecting and initial guess of the spatial optical properties of the
7 tissue;
8 (d) predicting the energy emerging from the tissue using an equation
9 of radiative transfer in an iterative process, wherein the prediction is a function of the

initial guess and a refraction index mismatch at a boundary of the tissue, and the iterative process generates a plurality of intermediate predictions;

(e) generating an objective function based on a normalized comparison of the prediction with the measured energy emerging from the scattering medium;

(f) generating a gradient of the objective function by adjoint differentiation;

(g) modifying the initial guess of the spatial properties of the tissue based on the gradient of the objective function;

(h) repeating steps (d) through (g) until at least one of a threshold of modifications to the initial guess is reached and the objective function reaches a threshold; and

(j) generating an image representation of the spatial optical properties of the tissue.

29. A system for reconstructing an image of a scattering medium, comprising:
a source for directing energy into the scattering medium at source location on the scattering medium;
a detector for measuring the energy emerging from the scattering medium at a detector location on the scattering medium;
an initial guess of internal properties of the scattering medium;
means for predicting the energy emerging from the scattering medium using an equation of radiative transfer, wherein the prediction is a function of the initial guess;

9 means for generating an objective function based on a comparison of the
10 prediction with the measurement;
11 means for generating a gradient of the objective function by a method of adjoint
12 differentiation;
13 means for modifying the initial guess of the properties of the scattering medium
14 based on the gradient of the objective function; and
15 means for generating an image representation of the internal properties of the
16 scattering medium.

1 30. The system according to claim 1, further comprising means for repeating
2 the predicting of the energy emerging from the scattering medium based on the modified
3 initial guess, generating the objective function and modifying the initial guess, until at
4 least one of a predetermined number of repetitions has occurred and the objective
5 function reaches a predetermined threshold.

1 31. The system according to claim 1, wherein the prediction depends on the
2 boundary conditions.

1 32. The system according to claim 31, wherein the boundary conditions
2 account for a refractive mismatch at an interface between the medium and at least one of
3 the detectors and source.

1 33. The system according to claim 1, wherein the prediction comprises an
2 iterative process producing intermediate results.

1 34. The system according to claim 33, wherein the intermediate results of the
2 prediction are stored.

1 35. The system according to claim 34, wherein generating the gradient of the
2 objective function by adjoint differences uses the intermediate results of the prediction.

1 36. The system according to claim 35, wherein generating the gradient
2 comprises stepping backward through the intermediate results of the prediction.

1 37. The system according to claim 1, wherein the equation of radiative
2 transfer is time independent.

1 38. The system according to claim 37, wherein the time independent equation
2 of radiative transfer is:

3
$$\omega \nabla \Psi(\mathbf{r}, \omega) + (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega) = S(\mathbf{r}, \omega) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega') d\omega'$$

4 where $\Psi(\mathbf{r}, \omega)$ is the radiance at the spatial position \mathbf{r} directed into a unit solid
5 angle ω , $S(\mathbf{r}, \omega)$ is the energy directed into the medium at spatial position \mathbf{r} into a unit
6 solid angle ω , μ_s is the scattering coefficient, μ_a is the absorption coefficient and $p(\omega, \omega')$
7 is the scattering phase function.

$$p(\cos\theta) = \frac{1-g}{2(1+g^2-2g\cos\theta)^{3/2}}$$

1 41. The system according to claim 40, wherein the time dependent equation of
2 radiative transfer is:

$$3 \quad \frac{1}{c} \frac{\partial \Psi(r, \omega, t)}{\partial t} = S(r, \omega, t) - \omega \cdot \nabla \Psi(r, \omega, t) - (\mu_a + \mu_s) \Psi(r, \omega, t) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(r, \omega', t) d\omega'$$

1 42. The system according to claim 41, wherein the scattering phase function
2 is:

$$3 \quad p(\cos\theta) = \frac{1-g^2}{2(1+g^2-2g\cos\theta)^{3/2}}$$

1 46. The system according to claim 1, wherein the objective function is:

1 47. The system according to claim 1, further comprising minimizing the
2 objective function.

1 48. The system according to claim 47, wherein minimizing the objective
2 function includes a one dimensional line search.

1 49. The system according to claim 48, wherein the one dimensional line
2 search is performed along a direction of the gradient of the objective function.

1 50. The system according to claim 49, wherein the one dimensional line
2 search is performed along a gradient-dependent direction.

1 51. The system according to claim 50, wherein the energy comprises near
2 infra-red energy.

1 52. The system according to claim 1, wherein the scattering medium contains
2 regions wherein the scattering coefficients are not substantially greater than the
3 absorption coefficients.

1 53. The system according to claim 1, wherein the scattering medium contains
2 a low scattering region embedded in a high scattering region.

1 54. The system according to claim 1, wherein the predicted energy is
2 determined using finite element methods.

1 55. The system according to claim 1, wherein the predicted energy is
2 determined using finite difference methods.

1 56. A system for imaging the spatial distribution of optical properties of
2 tissue, comprising:

3 (a) a source for directing energy into the scattering medium at a source
4 location on the tissue;

5 (b) a detector for measuring the energy emerging from the scattering medium
6 at a detector location on the tissue;

7 (c) an initial guess of spatial optical properties of the tissue;

8 (d) means for predicting the energy emerging from the tissue using an
9 equation of radiative transfer in an iterative process, wherein the prediction is a function
10 of the initial guess and a refraction index mismatch at a boundary of the tissue, and the
11 iterative process generates a plurality of intermediate predictions;

12 (e) means for generating an objective function based on a normalized
13 comparison of the prediction with the measured energy emerging from the scattering
14 medium;

15 (f) means for generating a gradient of the objective function by adjoint
16 differentiation;

17 (g) means for modifying the initial guess of the spatial properties of the tissue
18 based on the gradient of the objective function;
19 (h) means for repeating steps (d) through (g) until at least one of a threshold
20 of modifications to the initial guess is reached and the objective function reaches a
21 threshold; and
22 (j) means for generating an image representation of the spatial optical
23 properties of the tissue.

1 57. Computer executable software code stored on a computer readable
2 medium, the code for reconstructing an image of a scattering medium, comprising:
3 code to direct energy into the scattering medium at a source location on
4 the scattering medium;
5 code to measure the energy emerging from the scattering medium at a
6 detector location on the scattering medium;
7 code to receive an initial guess of internal properties of the scattering
8 medium;
9 code to predict the energy emerging from the scattering medium using an
10 equation of radiative transfer, wherein the prediction is a function of the initial guess;
11 code to generate an objective function based on a comparison of the
12 prediction with the measurement;
13 code to generate a gradient of the objective function by a method of
14 adjoint differentiation;

15 code to modify the initial guess of the properties of the scattering medium
16 based on the gradient of the objective function; and
17 code to generate an image representation of the internal properties of the
18 scattering medium.

1 58. Computer executable software code stored on a computer readable
2 medium, the code for imaging the spatial distribution of optical properties of tissue,
3 comprising:

4 (a) code to direct energy into the scattering medium at a source location on the
5 tissue;

6 (b) code to measure the energy emerging from the scattering medium at a detector
7 location on the tissue;

8 (c) code to receive an initial guess of spatial optical properties of the tissue;

9 (d) code to predict the energy emerging from the tissue using an equation of
10 radiative transfer in an iterative process, wherein the prediction is a function of the initial
11 guess and a refraction index mismatch at a boundary of the tissue, and the iterative
12 process generates a plurality of intermediate predictions;

13 (e) code to generate an objective function based on a normalized comparison of
14 the prediction with the measured energy emerging from the scattering medium;

15 (f) code to generate a gradient of the objective function by adjoint differentiation;

16 (g) code to modify the initial guess of the spatial properties of the tissue based on
17 the gradient of the objective function;

18 (h) code to repeat steps (d) through (g) until at least one of a threshold of
19 modifications to the initial guess is reached and the objective function reaches a
20 threshold; and
21 (j) code to generate an image representation of the spatial optical properties of the
22 tissue.

1 59. A computer readable medium having computer executable software code
2 stored thereon, the code for reconstructing an image of a scattering medium, comprising:
3 code to direct energy into the scattering medium at a source location on
4 the scattering medium;
5 code to measure the energy emerging from the scattering medium at a
6 detector location on the scattering medium;
7 code to receive an initial guess of internal properties of the scattering
8 medium;
9 code to predict the energy emerging from the scattering medium using an
10 equation of radiative transfer, wherein the prediction is a function of the initial guess;
11 code to generate an objective function based on a comparison of the
12 prediction with the measurement;
13 code to generate a gradient of the objective function by a method of
14 adjoint differentiation;
15 code to modify the initial guess of the properties of the scattering medium
16 based on the gradient of the objective function; and

17 code to generate an image representation of the internal properties of the
18 scattering medium.

1 60. A computer readable medium having computer executable software code
2 stored thereon, the code for imaging the spatial distribution of optical properties of tissue,
3 comprising:

4 (a) code to direct energy into the scattering medium at a source location on the
5 tissue;

6 (b) code to measure the energy emerging from the scattering medium at a detector
7 location on the tissue;

8 (c) code to receive an initial guess of spatial optical properties of the tissue;

9 (d) code to predict the energy emerging from the tissue using an equation of
10 radiative transfer in an iterative process, wherein the prediction is a function of the initial
11 guess and a refraction index mismatch at a boundary of the tissue, and the iterative
12 process generates a plurality of intermediate predictions;

13 (e) code to generate an objective function based on a normalized comparison of
14 the prediction with the measured energy emerging from the scattering medium;

15 (f) code to generate a gradient of the objective function by adjoint differentiation;

16 (g) code to modify the initial guess of the spatial properties of the tissue based on
17 the gradient of the objective function;

18 (h) code to repeat steps (d) through (g) until at least one of a threshold of
19 modifications to the initial guess is reached and the objective function reaches a
20 threshold; and

21 (j) code to generate an image representation of the spatial optical properties of the
22 tissue.

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